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Solution

1. $\int 3^x \cdot 4^{2x}\, dx$ $\quad u = 2x$

$\int (3^x\, 4^{2x}) \cdot \frac{1}{2} du$ $\quad du = 2dx$, $\quad dx = \frac{1}{2} du$

$\int \frac{3^x\, 4^{2x}}{2} du$

$\int \frac{3^x\, 16^x}{2} du$

$\int \frac{48^x}{2} du$ $\quad x = \frac{u}{2}$ $\quad b = \frac{u}{2}$

$\int \frac{48^{u/2}}{2} du$

$\int \frac{48^{u/2}}{2} \cdot 2du$

$\int 48^{u/2}$

$\int 48^b\, db$

$\frac{48^b}{\ln 48}$

$\frac{48^{u/2}}{\ln 48}$

$\frac{48^{2x/2}}{\ln 48}$

$\boxed{\frac{48^x}{\ln 48} + C}$ A

8. $\int \left(e^t + \frac{2}{t}\right) dt$

$\int e^t dt \int \frac{2}{t} dt$

$e^t \int \frac{2}{t} dt$

$e^t + 2 \int \frac{1}{t} dt$

$\boxed{e^t + 2\ln|t| + C}$ A

2nd Interval

$a^2 - x^2 = t^2$

$-2x\,dx = 2t\,dt$

$t = \sqrt{a^2 - x^2}$

2. $\int \left(\frac{3^x + 2^x}{5^x}\right) dx$

$\int \frac{3^x}{5^x} + \frac{2^x}{5^x}\, dx$

$\int \frac{3^x}{5^x} dx + \int \frac{2^x}{5^x} dx$

$\frac{\frac{3^x}{5^x}}{\ln \frac{3}{5}} + \frac{\frac{2^x}{5^x}}{\ln \frac{2}{5}}$

$\boxed{\frac{\left(\frac{3}{5}\right)^x}{\ln 3/5} + \frac{\left(\frac{2}{5}\right)^x}{\ln 2/5}}$ B

5. $\int \frac{dx}{x + \sqrt{x}}$

$= \int \frac{1}{(\sqrt{x}+1)\sqrt{x}} dx$ $\quad u = \sqrt{x} + 1$

$\quad du = \frac{1}{2\sqrt{x}} dx$

$\int \frac{1}{(\sqrt{x}+1)\sqrt{x}} \cdot 2\sqrt{x}\, du$ $\quad dx = 2\sqrt{x}\, du$

$\int \frac{1}{2(\sqrt{x}+1)}$

$2 \int \frac{1}{u}$

$= 2\ln u$

$\boxed{= 2\ln(\sqrt{x}+1) + C}$ C

9. $\int \sqrt{\frac{a-x}{a+x}}\, dx$

$\int \sqrt{\frac{a-x}{a+x} \cdot \frac{a-x}{a-x}}\, dx$

$\int \frac{a-x}{\sqrt{a^2-x^2}}\, dx$

$\int \frac{a\,dx}{\sqrt{a^2-x^2}} - \int \frac{x\,dx}{\sqrt{a^2-x^2}}$

$= a\sin^{-1}\left(\frac{x}{a}\right) + \int \frac{t\,dt}{t}$

$= a\sin^{-1}\left(\frac{x}{a}\right) + t + C$

$\boxed{= a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2-x^2} + C}$ A

3. $\int \frac{2\sin x\, dx}{\sin 2x}$

$\int \frac{2\sin x}{2\sin x \cos x} dx$

$\int \frac{1}{\cos x} dx$

$\int \sec x\, dx$

$\boxed{= \ln|\tan x + \sec x| + C}$ C

6. $\int \frac{1}{x}\left(\sqrt[6]{x^5}\right) dx$

$\int \frac{1}{x} (x^5)^{\frac{1}{6}} dx$

$\int \frac{1}{x} (x^{5/6})\, dx$

$\int x^{-1+5/6}\, dx$

$\int x^{\frac{-6+5}{6}}\, dx$

$\int x^{-1/6}\, dx$

$\frac{x^{-1/6+1}}{-1/6+1}$

$\frac{x^{5/6}}{5/6}$

$\boxed{\frac{6}{5} x^{5/6} + C}$ C

10. $\int x(3x^2-2)^6\, dx$ $\quad u = 3x^2 - 2$

$\int x(3x^2-2)^6 \cdot \frac{1}{6x} du$ $\quad du = 6x\,dx$

$\quad dx = \frac{1}{6x} du$

$\int \frac{(3x^2-2)^6}{6} du$

$\int \frac{u^6}{6} du$

$\frac{1}{6} \int \frac{u^6}{6}$

$\frac{1}{6}\left(\frac{u^7}{7}\right)$

$= \frac{u^7}{42}$

$\boxed{= \frac{(3x^2-2)^7}{42} + C}$ C

11. $\int \frac{dx}{x+\sqrt{x}}$

$\int \frac{1}{(\sqrt{x}+1)\sqrt{x}}\,dx$ $\quad u=\sqrt{x}+1$

$\int \frac{1}{(\sqrt{x}+1)\sqrt{x}} \cdot 2\sqrt{x}\,du$ $\quad du=\frac{1}{2\sqrt{x}}dx$ $\quad dx=2\sqrt{x}\,du$

$2\int \frac{1}{\sqrt{x}+1}$

$2\int \frac{1}{u}$

$2\ln(u)$

$\boxed{2\ln(\sqrt{x}+1)+C}$ D

12. $\int ((x^2+1)^3)\,dx$

$\int x^6+3x^4+3x^2+1$

$\left(\frac{x^{6+1}}{6+1}\right)+3\left(\frac{x^{4+1}}{4+1}\right)+3\left(\frac{x^{2+1}}{2+1}\right)+x$

$\frac{x^7}{7}+3\frac{x^5}{5}+3\frac{x^3}{3}+x+C$

$\boxed{\frac{x^7}{7}+3\frac{x^5}{5}+x^3+x+C}$ A

13. $\int \frac{x-8}{16x-x^2}\,dx$ $\quad u=16x-x^2$

$\int \frac{x-8}{16x-x^2}\cdot\frac{1}{16-2x}du$ $\quad du=16-2x\,dx$ $\quad dx=\frac{1}{16-2x}du$

$\int \frac{x-8}{16x-x^2}\cdot\frac{1}{2(8-x)}\,du$

$\int \frac{1}{-2(-x^2+16x)}du$

$\int \frac{1}{2u}\,du$

$\frac{1}{2}\ln(u)$

$\boxed{\frac{1}{2}\ln(16x-x^2)+C}$ B

14. $\int \frac{x^4\,dx}{x^2+1}$

$\int x^2-1+\frac{1}{x^2+1}\,dx$

$\int x^2dx+\int -1\,dx+\int\frac{1}{x^2+1}dx$

$\frac{x^3}{3}-x+\int\frac{1}{x^2+1}\,dx$

$\boxed{\frac{x^3}{3}-x+\arctan(x)+C}$ D

17. $\int \frac{(x-2)\,dv}{x^2+4x+4}$ $\quad u=x+2$ $\quad du=1dv$

$\int \frac{(x-2)\,dx}{(x+2)^2}$

$\int \frac{(u-4)}{u^2}\,du$

$\int \frac{1}{u}-\frac{4}{u}\,du$

$\int \frac{1}{u}du-\int\frac{4}{u}\,du$

$=\ln(u)+\frac{4}{u}+C$

$\boxed{=\ln(x+2)+\frac{4}{x+2}+C}$ B

18. $\int \frac{dx}{1-2x}$ $\quad u=1-2x$

$\int \frac{dx}{1-2x}\cdot -\frac{1}{2}du$ $\quad du=-2\,dx$ $\quad dx=-\frac{1}{2}du$

$-\frac{1}{2}\int\frac{dx}{u}\,du$

$-\frac{1}{2}\ln(u)$

$\boxed{-\frac{1}{2}\ln|1-2x|+C}$ letter D.

19. $\int \frac{dx}{x^3\sqrt{x^2-4}}$ $\quad u=\sqrt{x^2-4}$

$\int \frac{du}{(u^2+4)^2}$ $\quad dx=\frac{\sqrt{x^2-4}}{x}du$

$a=1,\ b=4,\ n=2$

$\int\frac{1+u}{(au^2+b)}\cdot\frac{2u-3}{2b(u-1)}\left(\frac{1}{(u^2+b)}\right)du$

$+\frac{u}{2b(n-a)(b^2-b)}$

$=\frac{u}{8(u^2+4)}+\frac{\arctan\left(\frac{u}{2}\right)}{16}$

$\frac{\sqrt{x^2-4}}{8x^2}-\frac{\arcsin\left(\frac{2}{x}\right)}{16}+C$

C $\boxed{\frac{2\sqrt{x^2-4}}{x^2}+\arcsin\frac{2}{x}+C}$

20. $\int \frac{e^x\sqrt{e^x-1}\,dx}{e^x+3}$ $\quad u=e^x$ $\quad du=e^x dx$

$\int\frac{u\sqrt{u-1}}{u+3}\,du$

$\int\frac{\sqrt{u-1}}{3}\,du$

$\frac{1}{3}\int(u-1)^{1/2}\,du$

$\frac{1}{3}\left(\frac{(u-1)^{3/2}}{3/2}\right)du$

$\frac{2(u-1)^{3/2}}{3}+C$

$\boxed{\frac{2(e^x-1)^{3/2}}{3}+C}$ C

21. $\int \frac{e^{5\ln x} - e^{4\ln x}}{e^{3\ln x} - e^{2\ln x}}\, dx$

$\int \frac{e^{4\ln x}(e^{\ln x}-1)}{e^{2\ln x}(e^{\ln x}-1)}\, dx$

$\int e^{2\ln x}\, dx \quad u = 2\ln x$

$\int e^{2\ln x} \cdot \frac{x}{2}\, du \quad dx = \frac{x}{2}\, du$

$\int \frac{e^{2\ln x}}{2} x\, du \quad t = \frac{3u}{2}$

$\int \frac{x^3}{2}\, du$

$\int \frac{(e^{u/2})^3}{2}\, du$

$\frac{1}{2}\int (e^{u/2})^3\, du$

$\frac{1}{2}\int \frac{2e^t}{3}\, dt$

$\frac{1}{2}\times\frac{2}{3}\int e^t\, dt$

$\frac{1}{3}\int e^t\, dx$

$\frac{e^t}{3} = \frac{e^{3u/2}}{3}$

$\frac{e^{\frac{3(2\ln x)}{2}}}{3}$

$\boxed{\frac{e^{3\ln x}}{3} + C}$

22. $\int \frac{x^3}{\sqrt{1-x}}\, dx \quad u = 1-x$

$\int \frac{x^3}{\sqrt{1-x}} \cdot -1\, du \quad du = dx$

$\int \frac{-x^3}{\sqrt{1-x+1}}\, du$

$\int -\frac{x^3}{\sqrt{1-x}}\, du \quad x = 1-u$

$\int -\frac{(1-u)^3}{\sqrt{u}}\, du$

$-\int \frac{(1-u)^3}{\sqrt{u}}\, du$

$-\int \frac{1-3u+3u^2-u^3}{\sqrt{u}}$

$-\int \frac{1}{\sqrt{u}} - \frac{3u}{\sqrt{u}} + \frac{3u^2}{\sqrt{u}} - \frac{u^3}{\sqrt{u}}\, du$

$-\int \frac{1}{u^{1/2}} - \frac{3u}{u^{1/2}} + \frac{3u^2}{u^{1/2}} - \frac{u^3}{u^{1/2}}\, du$

$-\int \frac{1}{u^{1/2}} - 3u^{1-\frac{1}{2}} + 3u^{2-\frac{1}{2}} - u^{3-\frac{1}{2}}\, du$

$-\int \frac{1}{u^{1/2}} - 3u^{\frac{1}{2}} + 3u^{3/2} - u^{5/2}\, du$

$-\int \frac{1}{u^{1/2}}\, du - 3\int u^{\frac{1}{2}}\, du + 3\int u^{3/2}\, du - \int u^{5/2}\, du$

$-\left(u^{\frac{1}{2}}\cdot 2 - 3\cdot\frac{2u^{3/2}}{3} + 3\cdot\frac{2u^{5/2}}{5} - \frac{2u^{7/2}}{7}\right)$

$-2u^{1/2} + 2u^{3/2} - \frac{6u^{5/2}}{5} + \frac{2u^{7/2}}{7}$

$\boxed{-2(1-x)^{1/2} + 2(1-x)^{3/2} - \frac{6(1-x)^{5/2}}{5} + \frac{2(1-x)^{7/2}}{7} + C}$

24. $\int \frac{\cos x}{\csc^{2020} x}\, dx$

$\int \cos x \cdot \sin^{2020} x\, dx$

$= \int \cos^{2020} x \sin\, dx$

$= \int \cos^{2020} x\, dx$

$\boxed{= \left(\frac{\sin^{2021}}{2021}\right) + C}$

25. $\int (\log_u u)\, dc$

$\log_u (u) \cdot c$ we can use $\int a\, dx = a \cdot x$

$1 \cdot c$

$\boxed{= c + k}$ k is the constant of integration

23. $\int \frac{(a^x + b^x + c^x)^2}{a^x b^x c^x}\, dx$